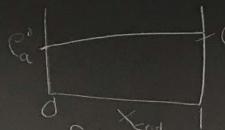


Exercise 5.3

Discussion of RAF


 $\Delta H_f^{\circ}(T_m) = X_c^W$
 $\frac{\Delta H_f^{\circ}(T_m)}{\Delta H_f^{\circ}(T_m')} = X_c^W$
 $100\% \text{ crystalline fraction}$
 $X_c = \frac{P_c(\text{actual sample})}{P_c^0}$
 $P = P_a + X_c(P_c - P_a)$
 $X_c = \frac{P - P_a}{P_c - P_a}$

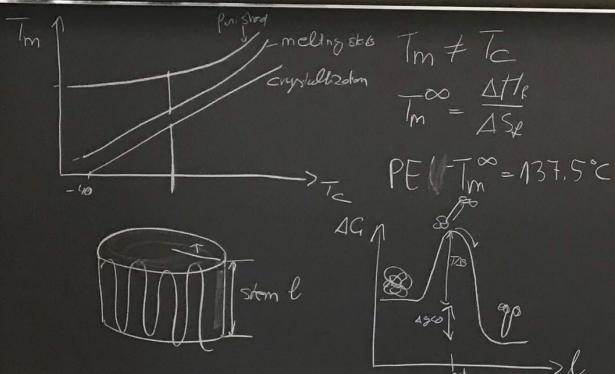
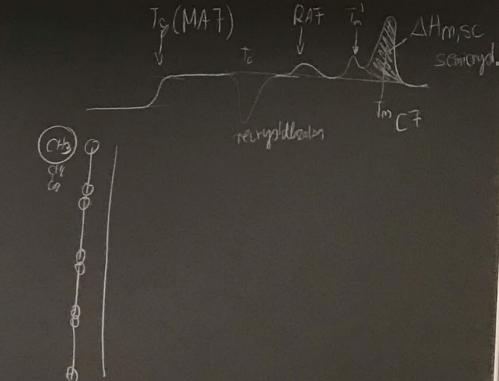
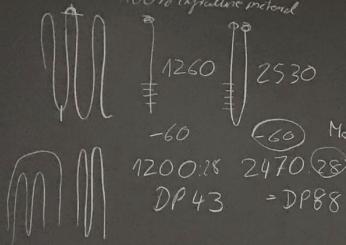
$2 \times \text{single} = \text{double}$

$2 \times 43 = 86$

difference of 2 units = loop

$43 \times 253 \text{ \AA} = 108.79 \text{ \AA}$

$\text{expt. value } 105 \text{ \AA}$



$\Delta G = \Delta G^{\infty} + \Delta S_v^{\infty} \text{ (1)}$

$\Delta G^{\infty} = -[2\pi r^2 + 2\pi r l] \gamma$

↑
surface destroyed by melting

$\Delta G^{\infty} = \pi r^2 l \Delta G_v^{\infty} \text{ (2)}$

∞ crystal of infinite size

$\Delta G_v^{\infty} = \Delta H_v^{\infty} - T_c \Delta S_v^{\infty}$
 $\rightarrow \Delta S_v^{\infty} = \frac{\Delta H_v^{\infty}}{T_m^{\infty}}$

$\Delta G_v^{\infty} = \Delta H_v^{\infty} \left(1 - \frac{T}{T_m^{\infty}} \right) \text{ (3)}$

$\text{for } T \rightarrow T_m^{\infty} \Delta G_v^{\infty} = 0$

$\Delta G = \left(\pi r^2 l \right) \Delta H_v^{\infty} \left(1 - \frac{T}{T_m^{\infty}} \right) - 2\pi r^2 \gamma \left(1 + \frac{l}{r} \right)$

$\xrightarrow{l \rightarrow 0} \left(\frac{T_m^{\infty} - T_m}{T_m^{\infty}} \right) 2\pi r^2 \gamma \left(1 + \frac{l}{r} \right)$

$\Delta T = \frac{2 \gamma}{\Delta H_v^{\infty}} \left(1 + \frac{l}{r} \right) T_m^{\infty}$

$\xrightarrow{r \rightarrow \infty} \Delta T = 2 \frac{T_m^{\infty}}{\Delta H_v^{\infty}} \frac{\gamma}{l}$

Derivation of Thomson Gibbs equation

Avrami equation



radial growth is

point x is reached if

number of nuclei in plane N per unit area

$$\text{average number of fronts} \bar{F} = \pi (\bar{r}t)^2 N \quad \bar{F} = \bar{F}^0 = 1$$

$$\text{distribution of } \bar{F} \quad P(\bar{F}) = \frac{e^{-\bar{F}}}{\bar{F}!} \quad \bar{F}^0 = 1$$

$$\text{if } \bar{F} = 0 \quad P(0) = \exp(-\bar{F})$$

= pure amorphous part

$$x_a = 1 - x_c$$

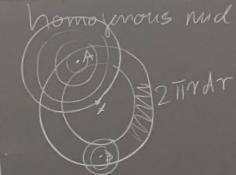
$$1 - x_c = P(0) = \exp(-\frac{t}{\bar{F}})$$

$$\ln \frac{1}{1 - x_c} = \frac{t}{\bar{F}} = \bar{r} \bar{r}^2 N t^2$$

$$x_c = 1 - \exp(-\bar{r} \bar{r}^2 N t^2)$$

short t: $x_c = 0$

long t: $x_c = 1$



$$x_c = 1 - \exp(-K t^m)$$

$$\ln \left(\ln \frac{1}{1 - x_c} \right) = \text{general Avrami eqn}$$

frequency of spontaneous nucleation \dot{N}

rate of formation $\dot{N} 2 \bar{r} \bar{r} dr$

radial growth is

$$1) d\bar{F} = (\dot{N} 2 \bar{r} \bar{r} dr)(t - \frac{r}{\bar{r}})$$

$$2) \bar{F} = 2 \bar{r} \bar{r} \int_{r=0}^{r=\bar{r}t} dr + (t - \frac{r}{\bar{r}}) dr$$

$$\int_{r=0}^{r=\bar{r}t} dr = \bar{F} = \frac{1}{3} \bar{r} \bar{r}^2 N \bar{r}^2 t^3 \quad \bar{F} = \ln \left(\frac{1}{1 - x_c} \right)$$

$$\ln \left(\frac{1}{1 - x_c} \right) = \frac{1}{3} N \bar{r}^2 t^3 \quad \Rightarrow x_c = 1 - \exp \left(-\frac{1}{3} N \bar{r}^2 t^3 \right)$$